$$\epsilon = 15\nu \left(\frac{\overline{\partial U}}{\partial \tau}\right)^2 / U^2$$

and the Kolmogorov frequency is

$$N_K = \epsilon^{1/4} \nu^{-3/4} U/2\pi.$$

The calculations give $\epsilon = 18.7\ \mathrm{cm}^2/\mathrm{sec}^3$ and $N_K = 56.4$ Hz.

The following conclusions can be drawn from our measurements under natural conditions: The automated measurement system is reliable in service; the software developed for it can be used to compute a large set of statistical parameters characterizing the fine structure of oceanological processes; the programs operate reasonably economically and efficiently; measurements of the one- and two-point moments indicate that the bottom zone in the experimental test region can be treated as a fully developed boundary layer, on which are superimposed modulated polycyclic processes, which can be filtered out in two-dimensional spectral processing.

NOTATION

H, probe submersion depth; λ_τ, Taylor time microscale; λ_U, Taylor space microscale; Λ_U, velocity macroscale; δ, boundary layer thickness; ϵ, dissipation function; N_K, Kolmogorov function; R_{UU}, $R_{U_1U_1}$, R_{tt}, autocorrelation functions; Indices: CA, HA, RT, measurements performed with a conduction anemometer, hot-wire anemometer, and resistance thermometer, respectively.

LITERATURE CITED

1. I. L. Povkh, Aerodynamic Experiments in Mechanical Engineering [in Russian], Mashinostroenie, Leningrad (1974).
2. J. O. Hinze, Turbulence, McGraw-Hill, New York (1959).
3. D. I. Grinval'd, "Some results of natural investigations of riverbed flows," in: Turbulent Flows [in Russian], Nauka, Moscow (1976).
4. L. S. G. Kovasznay, "Structure of the turbulent boundary layer," Phys. Fluids, 10, No. 9/11, 525-530 (1967).

DIFFUSION AND MIXING OF PASSIVE IMPURITIES IN A LINEAR VELOCITY FIELD

V. P. Kabashnikov and A. A. Kurskov

UDC 533.73:532.517.4

It is shown that the mixing process is described by a Lagrangian Green's function. The latter is obtained at both the dynamical and the statistical levels, which permits application of the results to turbulent media.

The question of the mixing of substances down to the molecular level occupies a special position in the problem of chemical reactions in turbulent media [1]. The mixing of substances is conditioned by molecular diffusion, which can play an appreciable role only at distances on the order of the internal scale of turbulence, where a linear dependence of relative velocities on distance holds.

The dynamical problem of diffusion of a scalar passive impurity in an unbounded linear velocity field was investigated in [2-5]. A study of the statistical characteristics of the field of a passive impurity in a medium with linear velocities was the subject of [6-8], where the structure function and short-wavelength asymptotic form of the concentration spectrum were obtained. A number of precise analytical results, particularly the spectrum of

---

Institute of Physics, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 43, No. 6, pp. 924-931, December, 1982. Original article submitted October 23, 1981.

0022-0841/82/4306-1343$07.50

fluctuation of the passive scalar in the dissipation region and its distribution function, were obtained in [9] for a velocity field δ-correlated in time. The results of a theoretical investigation of the process of mixing of spatially separated substances are given in the present article. First we consider the dynamical problem of diffusion in a medium whose velocity depends linearly on the coordinates, but without any restrictions on the form of the velocity-gradient tensor $\delta_{ij}(t)$, in contrast to [2-9]. It is shown that the result of the mixing (the integral over space of the product of the concentrations of the substances being mixed) depends only on the sum of the coefficients of diffusion and is described using a Green's function which depends on the Lagrangian coordinates. The very process of mixing can be interpreted as the "drifting" in Lagrangian coordinates of a cloud of one of the substances, to which the total coefficient of diffusion is assigned, onto the initial distribution of the other substance.

In the third part, devoted to statistical averaging, an equation is derived for the average concentration and average Lagrangian Green's function under the assumption that the time dependence of the velocity is a random, Gaussian, δ-correlated process. In addition, expressions are given for the tensor of dispersion of the impurity cloud in Lagrangian coordinates, and the characteristic times of mixing of arbitrary volumes of impurities are estimated. Estimates of the region of applicability of the results obtained are given in the conclusion.

Analysis of the Dynamical Problem. Let the arbitrary distributions of the concentrations of two substances, $c_1^0(\mathbf{x})$ and $c_2^0(\mathbf{x})$, with unit masses, be assigned in a turbulent medium at the initial time.

The dynamic equation describing the variation of a concentration $c(\mathbf{x}, t)$ in a moving incompressible liquid has the form

$$\frac{\partial c(\mathbf{x}, t)}{\partial t} + v_i(\mathbf{x}, t)\frac{\partial c(\mathbf{x}, t)}{\partial x_i} = \mu \Delta c(\mathbf{x}, t), \tag{1}$$

where $\mathbf{v}(\mathbf{x}, t)$ is the velocity of the liquid; and μ, coefficient of molecular diffusion. At distances on the order of or less than the internal scale of turbulence, the liquid velocity depends linearly on the coordinates [10]:

$$v_i(\mathbf{x}, t) = \sigma_{ij}(t) x_j. \tag{2}$$

The matrix $\|\sigma(t)\|$ in a turbulent liquid is a random function of time.

We change from the Eulerian coordinate system $\mathbf{x}$ to the Lagrangian system $\mathbf{X}$,

$$X_i = a_{ij}(t) x_j, \tag{3}$$

where the elements of the matrix $\|a\|$ satisfy the equation

$$a_{ij}(t) = -a_{ik}\sigma_{kj}, \quad a_{ij}(0) = \sigma_{ij}. \tag{4}$$

In the Lagrangian coordinate system Eq. (1) has the form

$$\frac{\partial c(\mathbf{X}, t)}{\partial t} = \mu a_{ij} a_{kj} \frac{\partial^2 c(\mathbf{X}, t)}{\partial X_i \partial X_k}. \tag{5}$$

The solution of Eq. (5) can be written in the form

$$c(\mathbf{X}, t) = \int \mathcal{G}(\mathbf{X} - \mathbf{Y}, t)\, c^0(\mathbf{Y})\, d\mathbf{Y}, \tag{6}$$

where $\mathcal{G}$ is the so-called Lagrangian of Green's function, describing how a substance diffuses in the Lagrangian coordinate system if its initial concentration distribution had the form of a δ-function:

$$\mathcal{G}(\mathbf{R}, t) = (2\pi)^{-3} \int \exp\{ik_j R_j - \mu b_{jn} k_j k_n\}\, d\mathbf{k}. \tag{7}$$

The matrix $\|b\|$, the elements of which satisfy the equation

$$\dot{b}_{jn}(t) = a_{jk} a_{nk}, \quad b_{jn}(0) = 0, \tag{8}$$

is symmetrical, positive definite, and depends only on the deformational part of the transformation (3).

Using (3), (6), and (7), we find the solution of Eq. (1):

$$c(\mathbf{x}, t) = \int G(\mathbf{x}, t|\mathbf{y})\, c^{0}(\mathbf{y})\, d\mathbf{y}, \tag{9}$$

where G is the Green's function of Eq. (1), henceforth called the Eulerian equation, since it describes diffusion in the Eulerian coordinate system:

$$G(\mathbf{x},\ t|\mathbf{y}) = (2\pi)^{-3} \int \exp\{-ik_j y_j + ia_{jn} k_j x_n - \mu b_{jn} k_j k_n\}\, d\mathbf{k}. \tag{10}$$

The Lagrangian and Eulerian of Green's function are connected by the relation

$$\mathcal{G}(\mathbf{R},\ t) = G(0,\ t|\mathbf{R}). \tag{11}$$

We introduce the Richardson function $\psi(\mathbf{x},\ t)$ [10] for the concentrations c_1 and c_2 of the substances:

$$\psi(\mathbf{x},\ t) = \int c_1(\mathbf{y},\ t)\, c_2(\mathbf{y} + \mathbf{x},\ t)\, d\mathbf{y}, \tag{12}$$

which in the velocity field (2) satisfies an equation analogous to (1):

$$\frac{\partial \psi}{\partial t} + v_i \frac{\partial \psi}{\partial x_i} = (\mu_1 + \mu_2)\, \Delta\psi. \tag{13}$$

The Lagrangian of the Richardson function, $\Psi(\mathbf{X},\ t)$, defined as

$$\Psi(\mathbf{X},\ t) = \int c_1(\mathbf{Y},\ t)\, c_2(\mathbf{Y} + \mathbf{X},\ t)\, d\mathbf{Y}, \tag{14}$$

satisfies an equation of the type (5):

$$\frac{\partial \Psi}{\partial t} = (\mu_1 + \mu_2)\, a_{ij} a_{kj} \frac{\partial^2 \Psi}{\partial X_i\, \partial X_k}. \tag{15}$$

Setting $c_1 = c_2$ in (12), we can introduce the tensor of dispersion of the Eulerian dimensions r_{ij} of the cloud of diffusing impurity:

$$r_{ij} = \int x_i x_j \psi(\mathbf{x},\ t)\, d\mathbf{x}, \tag{16}$$

equal to twice the moment $\overline{(x_i - \bar{x}_i)(x_f - \bar{x}_j)}$ of the density distribution of the impurity cloud relative to its center of gravity $\bar{\mathbf{x}} = \int \mathbf{x} c(\mathbf{x})\, d\mathbf{x}$.

Similarly, we introduce the dispersion tensor for the Lagrangian dimensions R_{ij} of the cloud:

$$R_{ij} = \int X_i X_j \Psi(\mathbf{X},\ t)\, d\mathbf{X} = 2\overline{(X_i - \bar{X}_i)(X_j - \bar{X}_j)}, \tag{17}$$

which, with the help of (15) and (8), can be written in the form

$$R_{ij}(t) = R_{ij}^0 + \int X_i X_j \mathcal{G}(\mathbf{X},\ t)\, d\mathbf{X} = R_{ij}^0 + 4\mu b_{ij}(t). \tag{18}$$

The degree of mixing of the two substances in the entire volume, determined by the value of $\psi(\mathbf{x},\ t)$ at $\mathbf{x} = 0$ can be written in the Eulerian (12) or Lagrangian (14) representations:

$$\psi(0,\ t) = \int c_1(\mathbf{y},\ t)\, c_2(\mathbf{y},\ t)\, d\mathbf{y} = \int c_1(\mathbf{Y},\ t)\, c_2(\mathbf{Y},\ t)\, d\mathbf{Y}. \tag{19}$$

The mixing of the substances is closely connected with diffusion in the Lagrangian coordinate system. Using the similarity of Eqs. (5) and (15), we can write the quantity $\psi(0,\ t)$ in a form analogous to (6):

$$\psi(0,\ t) = \int \mathcal{G}(\mathbf{X},\ t)|_{\mu_1+\mu_2} \Psi^0(\mathbf{X})\, d\mathbf{X} = \int c_1(\mathbf{X},\ t)|_{\mu_1+\mu_2} c_2^0(\mathbf{X})\, d\mathbf{X}. \tag{20}$$

In accordance with (20), the mixing process can be treated as the "drifting" in Lagrangian coordinates of the concentration distribution of a substance with a total coefficient of diffusion $c_1(\mathbf{X},\ t)|_{\mu_1+\mu_2}$ onto the initial concentration distribution $c_2^0(\mathbf{X})$ of the other substance.

Let the mixing substances be spatially separated at the initial time (i.e., $\psi(0,\ 0) = 0$) and the distance between the centers of gravity $\mathbf{x}_1^0$ and $\mathbf{x}_2^0$ of the distributions $c_1^0(\mathbf{x})$ and $c_2^0(\mathbf{x})$ equal R. By mixing time we shall understand the time in which appreciable overlapping of $c_1(\mathbf{x},\ t)$ and $c_2(\mathbf{x},\ t)$ or, by virtue of (20), $c_1(\mathbf{X},\ t)|_{\mu_1+\mu_2}$ and $c_2^0(\mathbf{X})$ occurs, which corresponds to to the maximum of the time dependence $\psi(0,\ t)$. It follows from Eq. (5) that diffusion does not alter the position of the center of gravity of the impurity cloud in the Lagrangian coordinate system. Then it is easy to see that pronounced overlapping of $c_1(\mathbf{X}, t)|_{\mu_1+\mu_2}$ and $c_2^0(\mathbf{X})$ occurs when the average Lagrangian dimension $c_1(\mathbf{X}, t)|_{\mu_1+\mu_2}$ of the cloud is comparable with the initial distance between the substances being mixed:

$$\overline{(\mathbf{X}_1 - \overline{\mathbf{X}}_1^0)^2} \approx R^2. \tag{21}$$

We note that in the case when the initial distributions of the substances had the form of δ-functions separated by a distance R, $\Psi^0(\mathbf{X}) = \delta(\mathbf{X} - \mathbf{R})$ and the degree of mixing in the entire volume is described, in accordance with (20), by the Lagrangian Green's function $\mathcal{G}(\mathbf{R}, t)|_{\mu_1+\mu_2}$.

Model of δ-Correlated Random Velocities. We assume that the matrix $\sigma_{ij}(t)$ represents a Gaussian, stationary, random process δ-correlated in time. Averaging Eq. (1) over realizations of the velocity field using the method of [11], a detailed exposition of which is contained in [12] and [13], we obtain, allowing for the isotropy of the velocity field,

$$\frac{\partial}{\partial t}\langle c \rangle = D\left[2x^2\Delta - x_i x_j \frac{\partial^2}{\partial x_i \partial x_j}\right]\langle c \rangle + \mu\Delta\langle c \rangle, \tag{22}$$

where

$$D = \frac{2\pi}{15}\int_0^\infty k^2 E(k, \omega = 0)\, dk; \tag{23}$$

E(k, ω) is the spatial–temporal spectral density of turbulent energy; and <...> is an average over the ensemble of realizations of the velocity field. Because the differential operator on the right side of (22) is Hermitian, Eq. (11) takes the form

$$\langle \mathcal{G}(\mathbf{R}, t) \rangle = \langle G(0, t|\mathbf{R}) \rangle = \langle G(\mathbf{R}, t|0) \rangle. \tag{24}$$

Thus, $\langle\mathcal{G}\rangle$ satisfies Eq. (22) with the initial condition $c^0 = \delta(x)$. The solution of Eq. (22) and the asymptotic form of $\langle\mathcal{G}\rangle$ are given in the Appendix.

We obtain the equation for $\langle r_{ij} \rangle$ by using the moment $x_i x_j$ from (22) and allowing for the similarity between (1) and (13):

$$\langle \dot{r}_{ij} \rangle = 4\mu\delta_{ij} + D\,[4\delta_{ij}\langle r_{nn} \rangle - 2\langle r_{ij} \rangle]. \tag{25}$$

The system (25) has the solution

$$\langle r_{ij} \rangle = r_{ij}^0 \exp(-2Dt)\,(i \neq j), \tag{26}$$

$$\langle r_{ij} \rangle = r_{ij}^0 \exp(-2Dt) + \frac{2\mu}{5D}\left[\exp(10Dt) - 1\right] + \frac{r_{nn}^0}{3}\left[\exp(10Dt) - \exp(-2Dt)\right](i = j), \tag{27}$$

$$\langle r_{nn} \rangle = \overline{\langle (\mathbf{x} - \overline{\mathbf{x}})^2 \rangle} = r_{nn}^0 \exp(10Dt) + \frac{6\mu}{5D}[\exp(10Dt) - 1]. \tag{28}$$

A characteristic feature of diffusion in the Eulerian coordinate system is that the initial shape of the impurity cloud is rapidly left behind, even when μ = 0, because of the isotropy of the velocity field, as well as the exponentially rapid increase in the average size of the cloud [6, 12]. Equation (24) lets us write the dispersion tensor for the Lagrangian dimensions of the impurity cloud (17) in the form

$$\langle R_{ij} \rangle = R_{ij}^0 + \delta_{ij}\frac{2\mu}{5D}[\exp(10Dt) - 1], \tag{29}$$

$$\langle R_{nn} \rangle = R_{nn}^0 + \frac{6\mu}{5D}[\exp(10Dt) - 1]. \tag{30}$$

Diffusion in the Lagrangian coordinate system is characterized by spherization and an exponentially rapid increase in the size of the impurity cloud, taking place only for μ ≠ 0, however, which is connected with the elimination of the influence of the drift of a liquid particle over the average size of the cloud in the Lagrangian coordinate system.

Knowledge of the average Lagrangian radius of the cloud allows one to estimate through (21) the time of mixing two substances preliminarily separated by a distance R:

$$t_m = (10\,D)^{-1} \ln\left(1 + \frac{5D}{6\mu} R^2\right). \tag{31}$$

The region of applicability of the model of δ-correlated velocities can be illustrated on the example of a problem which admits of an exact solution without the assumption of δ-correlation.

Let us consider the increase in the Lagrangian size of an impurity cloud having an initial distribution in the form of a δ-function in a velocity field characterized by a velocity-gradient tensor

$$\sigma_{ij}(t) = \delta_{ij}\, a_i(t), \ \sum_{i=1}^{3} a_i = 0, \tag{32}$$

where $a_i(t)$ is a Gaussian, stationary, random process. Here and below there is no summation over the recurrent indices. Using (18), (4), and (8), we find

$$\langle R_{ii} \rangle = 2\mu \int_0^t \exp\left\{2 \int_0^\tau \int_0^\tau \langle a_i(t_1)\, a_i(t_2) \rangle\, dt_1 dt_2\right\} d\tau. \tag{33}$$

Repeating the operations based on the δ-correlation approximation, which result in (22), (25), and (29) but with the velocity field corresponding to (32), we obtain

$$\langle R_{ii} \rangle = \frac{\mu}{2A_i} (\exp(4\,A_i t) - 1), \tag{34}$$

where

$$A_i = \int_0^\infty \langle a_i(0)\, a_i(t) \rangle\, dt. \tag{35}$$

Equation (33) changes into (34) if the times under consideration markedly exceed the velocity-correlation time, which evidently is the main condition for applicability of the approximation of δ-correlated velocities to the mixing problem.

In conclusion, we estimate the region of applicability of the results obtained. In accordance with (23), the quantity D can be estimated as

$$D \approx \tau_* \frac{d^2}{dx^2} D_{LL}(x)|_{x=0} \approx \sqrt{\frac{\nu}{\varepsilon}}\, \frac{\varepsilon}{\nu} = \sqrt{\frac{\varepsilon}{\nu}} \approx \tau_l^{-1}, \tag{36}$$

where D_{LL} is the velocity structure function; and τ_*, velocity-difference correlation time, conforming in order of magnitude with the characteristic time of the internal scale $\tau_l \sim \nu^{1/2}\varepsilon^{-1/2}$.

The results obtained in the report are valid at distances much less than the internal scale $l \sim \nu^{3/4}\varepsilon^{-1/4}$ and at times longer than τ_l. It follows from (28) that an impurity cloud initially represented by a δ-function reaches a size of order l in a time

$$t_l \approx (10\,D)^{-1} \ln\left(1 + \frac{5D}{6\mu} l^2\right) \approx \tau_l \ln\left(1 + \frac{5\nu}{6\mu}\right). \tag{37}$$

For $\mu \gg \nu$ the time t_l is comparable with τ_l or less than it. In such times the influence of convective transfer on the diffusion of the cloud and mixing is not yet able to be felt.

In the opposite limiting case ($\mu \ll \nu$) the time for the cloud to reach the size $\sim l$ is $t_l \gg \tau_l$. In this case the approximation of δ-correlated velocities is valid. It follows from (28), (30), and (A.4) that for $\mu \ll \nu$ there is an interaction between diffusion and hydrodynamic motion, resulting in a sharp increase in the cloud expansion velocity and consderably intensifying the mixing process compared with the case of a stationary medium.

APPENDIX

In accordance with (24), the equation for the average Lagrangian Green's function coincides with Eq. (22) with an initial condition represented by a δ-function at the origin of coordinates. Since with a spherically symmetrical initial condition the solution of (22) depends only on the absolute value of the coordinate, for the average Lagrangian Green's function we obtain the equation

$$\dot{\mathcal{G}}(x,\ t) = 2(Dx^2+\mu)\left[\ \mathcal{G}'' + \frac{2\mathcal{G}'}{x}\ \right] - 2Dx^2\mathcal{G}'' \tag{A.1}$$

with the initial condition

$$\mathcal{G}(x,\ 0) = -\frac{\delta'(x)}{2\pi x}. \tag{A.2}$$

Equation (A.1) describes the diffusion in Lagrangian variables of a substance for which the coefficient of molecular diffusion equals 2μ.

Changing to the dimensionless variables

$$r = x\sqrt{\frac{D}{2\mu}},\quad \tau = Dt \tag{A.3}$$

in (A.1) and (A.2), separating the variables in (A.1), and using Fourier and Kantorovich–Lebedev [14] transformations, we obtain the solution of (A.1) with the initial condition (A.2) in the form

$$\mathcal{G}(r,\ \tau) = \left(\frac{D}{2\mu}\right)^{3/2}\frac{\exp\left(-\frac{9\tau}{4}\right)}{2\pi^4}\int\limits_0^\infty d\alpha\, \alpha \operatorname{sh}\pi\alpha \exp(-\alpha^2\tau)\times$$

$$\times\left|\Gamma^2\left(\frac{3}{4}+\frac{i\alpha}{2}\right)\right|^2 F\left(\frac{3}{4}+\frac{i\alpha}{2},\ \frac{3}{4}-\frac{i\alpha}{2},\ \frac{3}{2},\ -r^2\right), \tag{A.4}$$

where $F(a,\ b,\ c,\ z)$ is a hypergeometric function.

From (A.4) we can obtain the following asymptotic forms:

for $\tau \ll 1$

$$\mathcal{G}(r,\ \tau) = \left(\frac{D}{2\mu}\right)^{3/2}(4\pi\tau)^{-3/2}\exp\left(-\frac{r^2}{4\tau}\right); \tag{A.5}$$

for $\tau \gg 1,\ r \ll 1$

$$\mathcal{G}(r,\ \tau) = \left(\frac{D}{2\mu}\right)^{3/2}\frac{\Gamma^4\left(\frac{3}{4}\right)}{8\pi^{5/2}\,\tau^{3/2}}\exp\left(-\frac{9\tau}{4}\right); \tag{A.6}$$

for $\tau \gg 1,\ 1 \ll \ln r \ll \tau$

$$\mathcal{G}(r,\ \tau) = \left(\frac{D}{2\mu}\right)^{3/2}\frac{\Gamma_2\left(\frac{3}{4}\right)\exp\left(-\frac{9\tau}{4}\right)}{8\pi^{5/2}r^{3/2}\tau^{3/2}}\ln r\exp\left(-\frac{\ln^2 r}{4\tau}\right); \tag{A.7}$$

for $\tau \gg 1,\ \ln r \gg \tau$

$$\mathcal{G}(r,\ \tau) = \left(\frac{D}{2\mu}\right)^{3/2}\frac{\exp\left(-\frac{9\tau}{4}\right)}{8\pi^{5/2}r^{3/2}\tau^{3/2}}\ln r\exp\left(-\frac{\ln^2 r + 2\ln 2\ln r}{4\tau}\right). \tag{A.8}$$

NOTATION

a_{ij}, matrix of conversion from Eulerian to Lagrangian coordinates; $c(x,\ t)$, concentration of a substance at the point x at the time t; $c_1(X,\ t)|_{\mu_1+\mu_2}$, concentration of a substance characterized by a coefficient of molecular diffusion of $\mu_1 + \mu_2$; $E(k,\ \omega)$, spatial–temporal spectral density of turbulent energy; $\mathcal{G}(x,\ t)$, Green's function of Eq. (5); $G(x,t|y)$, Green's function of Eq. (1); l, internal scale of turbulence; r_{ij}, tensor of dimensions of cloud of diffusing impurity in the Eulerian coordinate system, defined by Eq. (16); R, initial distance between centers of gravity of concentration distributions of the mixing substances; R_{ij}, tensor of dimensions of the cloud of diffusing impurity in the Lagrangian coordinate system, defined by Eq. (17); t, time; t_l, time in which the Lagrangian size of the impurity cloud becomes equal to the internal space of turbulence; t_m, mixing time; $v_i(x,\ t)$, i-th Cartesian velocity component of the medium at the point x at the time t; x, y, Eulerian vectors; X, Y, Lagrgangian vectors; Δ, Laplace operator; ε, specific rate of dissipation of turbulent energy; μ, coefficient of molecular diffusion; ν, coefficient of kinematic viscosity; σ_{ij}, velocity-gradient tensor; τ_l, characteristic time of the internal scale of turbulence; $\psi(x)$, Richardson function (12); $\Psi(X)$, Lagrangian of Richardson function (14). Indices: i, j, k, numbering of Cartesian components (they take values of 1, 2, 3); ^{0}, initial value; $^-$, averaging over cloud density; $\cdot$, time derivative; $'$, derivative with respect to a coordinate.

LITERATURE CITED

1. V. Z. Kompaniets, A. A. Ovsyannikov, and L. S. Polak, Chemical Reactions in Turbulent Streams of Gas and Plasma [in Russian], Nauka, Moscow (1979).
2. A. A. Townsend, "The diffusion of heat spots in isotropic turbulence," Proc. R. Soc. London, Ser. A, 209, No. 1098, 418-430 (1951).
3. A. A. Townsend, "The diffusion behind a line source in homogeneous turbulence," Proc. R. Soc. London, Ser. A, 224, 1159, 487-512 (1954).
4. E. A. Novikov, "Turbulent diffusion in a stream with a transverse velocity gradient," Prikl. Mat. Mekh., 22, No. 3, 412-414 (1958).
5. D. E. Elric, "Source function for diffusion in uniform shear flow," Aust. J. Phys., 15, No. 3, 283-288 (1962).
6. G. K. Batchelor, "Small-scale variation of convected quantities like temperature in turbulent fluid. Part 1. General discussions and the case of small conductivity," J. Fluid Mech., 5, 113-133 (1959).
7. E. A. Novikov, "Fluctuations of electron density in the ionosphere," Dokl. Akad. Nauk SSSR, 139, No. 3, 587-589 (1961).
8. P. G. Saffman, "On the fine-scale structure of vector fields convected by a turbulent fluid," J. Fluid Mech., 16, No. 4, 545-572 (1963).
9. R. H. Kraichnan, "Convection of a passive scalar by a straining field," J. Fluid Mech., 64, No. 4, 737-762 (1974).
10. A. S. Monin and A. M. Yaglom, Statistical Fluid Mechanics [in Russian], Part 2, Nauka, Moscow (1967).
11. E. A. Novikov, "Functionals and the method of random forces in the theory of turbulence," Zh. Eksp. Teor. Fiz., 47, No. 5 (11), 1919-1926 (1964).
12. V. I. Klyatskin, Statistical Description of Dynamical Systems with Fluctuating Parameters [in Russian], Nauka, Moscow (1975).
13. V. I. Klyatskin, Statistical Equations and Waves in Randomly Inhomogeneous Media [in Russian], Nauka, Moscow (1980).
14. V. A. Ditkin and A. P. Prudnikov, Integral Transformations and Operator Calculus [in Russian], Nauka, Moscow (1974).